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Inhomogeneous coupled non-linear Schrödinger systems

Abstract: In this work, one considers the Cauchy problem for the inhomogeneous Schrödinger system with a coupled power-type non-linearity

$$\begin{cases} i\dot{u}_j + \Delta u_j = \sigma|x|^b\Big(\sum_{k=1}^{m} a_{jk}|u_k|^p\Big)|u_j|^{p-2}u_j; \\ u_j(0,.) = \psi_j \end{cases}$$

where $u_j : \mathbb{R}^N \times \mathbb{R} \to \mathbb{C}$ for $j \in [1,m]$ and $1 \leq N \leq 3$. The inhomogeneous term is $|.|^b$ for some $b < 0$ and the source term satisfy $p \geq 2$. The constant $\sigma = \pm 1$ refers to the attractive or repulsive regime and the coupling coefficients satisfy

$$a_{jk} = a_{kj} \geq 0 \quad \text{and} \quad a_{jj} > 0 \quad \text{for } j \in [1,m].$$

In the focusing sign, a sharp dichotomy of global existence and scattering versus finite time blow-up of solutions is obtained using some variational methods, a sharp Gagliardo-Nirenberg type inequality and a new approach due to Dodson-Murphy [Proc. Am. Math. Soc. 145(11), 4859-4867 (2007)]. In the defocusing sign, using a classical Morawetz estimate, the scattering of global solutions in the energy space is proved.