New regularity criterion for 3D Navier Stokes equations

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ABSTRACT

We study the three-dimensional periodic Navier-Stokes equations in Gevrey-Sobolev spaces. We start by proving the local in time well-posedness for arbitrary large in $H^{1/2}_{a,s}(\mathbb{T}^3)$ initial data. We also prove the global in time well-posedness provided that the initial data satisfies a smallness condition. Finally, we establish a non-standard global in time existence criteria, which is based on breaking up the Fourier sum $\sum_{k\in\mathbb{Z}^3} e^{\frac{a}{s}|k|^{1/s}}|\hat{u}(t,k)|$ of the solution u into low frequency modes up to m and high frequency modes down to m. In particular, we prove that there exists a wave-number m that depends on the viscosity and the $H^{1/2}_{a,s}(\mathbb{T}^3)$-norm of the initial data, such that if the map $t \mapsto F_m(t) = \sum_{|k|\leq m} e^{\frac{a}{s}|k|^{1/s}}|\hat{u}(t,k)| - \sum_{|k|>m} e^{\frac{a}{s}|k|^{1/s}}|\hat{u}(t,k)|$ keeps a constant sign on the time interval of existence $[0,T)$, then the global in time existence of the solution follows.

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