Title : Homoclinic solutions in $\mathbb{R}^2$ for singular hamiltonian systems with weak force potentials

Abstract : We study the existence of homoclinic solutions for a class of singular hamiltonian systems

$$\ddot{q}(t) + V'(q) = 0, \ t \in \mathbb{R}, \tag{HS}$$

where $q : \mathbb{R} \longrightarrow \mathbb{R}^2$ and $V \in C^2(\mathbb{R}^2 \setminus \{e\}, \mathbb{R})$ is a potential with a singularity at a point $e \in \mathbb{R}^2 \setminus \{0\}$. We consider V which behaves like $-\dfrac{1}{|q-e|^\alpha}$ as $q \to e$ with $\alpha \in]1, 2[$.

Under the assumption 0 is a strict global maximum for V, we establish the existence of homoclinic solution emanating from 0, i.e. a solution q of (HS) such that

$$q \neq 0 \ \text{ and } \ |q(t)| + |\dot{q}(t)| \longrightarrow 0 \ \text{ as } \ t \to \pm\infty.$$