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# On a fractional KPZ type equation including nonlocal gradient terms: the stationary case.

**Abstract:**

We obtain, first, a global fractional Calderón-Zygmund type regularity theory for the fractional Poisson problem. More precisely, for $\Omega \subset \mathbb{R}^N$, $N \geq 2$, a bounded domain with boundary $\partial\Omega$ of class $C^2$, $s \in (0,1)$ and $f \in L^m(\Omega)$ for some $m \geq 1$, we consider the problem

$$\begin{cases} (-\Delta)^s u = f\,, & \text{in } \Omega, \\ \qquad u = 0\,, & \text{in } \mathbb{R}^N \setminus \Omega, \end{cases} \tag{P}$$

and, according to $m$, we find the values of $s \leq t < \min\{1, 2s\}$ and of $1 < p < +\infty$ such that $u$ belongs to the Bessel potential space $L^{t,p}(\mathbb{R}^N)$ and the fractional Sobolev space $W^{t,p}(\mathbb{R}^N)$.

As an application, we prove new existence results for the following deterministic Kardar–Parisi–Zhang type problem involving non-local "gradient terms" :

$$\begin{cases} (-\Delta)^s u = \mu(x)\,|\mathbb{D}(u)|^p + \rho f(x), & \text{in } \Omega, \\ \qquad u = 0, & \text{in } \mathbb{R}^N \setminus \Omega, \end{cases} \tag{KPZ}$$

where $p > 1$ and $\rho > 0$ are real parameters, $f$ belongs to a suitable Lebesgue space, $\mu \in L^\infty(\Omega)$ and $\mathbb{D}$ represents a nonlocal "gradient term". Moreover, optimality results, for (KPZ), are also given.

**Key words:** *Fractional Laplacian, fractional Poisson equation, Calderón-Zygmund regularity, KPZ problem, nonlocal gradient.*

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